PHYSICS THROUGH TEACHING LAB – IV

Newton's Law of Cooling

D. A. DESAI
C-10 Patil Regency, Erandawana, Pune 411004
e-mail:dattatraya.desai@gmail.com

Introduction

When hot bodies are left in the open they are found to cool gradually. Newton found that the rate of cooling was proportional to the excess of temperature of the body over that of the surroundings. This observation is what is called Newton's law of Cooling. It is not known if Newton attempted any theoretical explanation of this phenomenon. But it is unlikely because the concepts about heat were not clear in those times. But what is important is that the original statement of the conditions for the validity of Newton's law included the presence of a draught.[1]

In the course of time Stefan's law of radiation was discovered and it explained how a hot body loses heat by radiation. According to this law every body radiates energy at a rate proportional to the fourth power of its absolute temperature T. Since the body absorbs energy radiated from the surroundings which is proportional to the fourth power of the absolute temperature T_0 of the environment and the constant of proportionality is the same, the net loss of heat by the body from radiation is proportional to the difference $(T^4 - T_0^4)$. Possibly, in an attempt to explain Newton's law of cooling from first principles, it occurred to somebody that

$$T^4 - T_0^4 = (T^2 + T_0^2)(T + T_0)(T - T_0)$$

and when T is approximately equal to T_0, the radiation loss from the body is approximately proportional to $T - T_0$ if this difference is very small compared to T_0. Newton's law of cooling was then considered to be a direct inference from Stefan's law of radiation. But two important points were overlooked. First is the fact that the presence of draught of air gave the observed effect and the second is that the approximation is justified if the neglected terms are really small. The first point makes it clear that the cooling is mainly due to a draught of air and not due to radiation alone. The second point is about the temperatures at which Newton's law is observed. To make this point clear let us write $T = T_0 + \Delta T$. Then

$$(T^4 - T_0^4) = (T_0^4 + 4\,T_0^3\,\Delta T + 6T_0^2 \Delta T^2 + 4T_0\,\Delta T^3 + \Delta T^4 - T_0^4)$$

$$= 4T_0^3\Delta t\left(1+\frac{3}{2}\frac{\Delta T}{T_0}+\left(\frac{\Delta T}{T_0}\right)^2+\left(\frac{\Delta T}{T_0}\right)^3\right)$$

To justify neglecting all terms with ΔT their value must be small in comparison to unity. Let the temperature of the surroundings be 27º C. As can be seen the neglected term increases by more than 0.01 for every 2 degree difference of temperature. So if the difference between the temperatures of the body and the surroundings is 50º C the neglected term is more than 0.25. This cannot be called negligible in comparison with unity. This shows that the approximation is invalid from the point of mathematics.

But what is important is that Stefan's law of radiation and Newton's law of cooling refer to quite different temperatures. Stefan's law refers to the temperature of the surface of the body and Newton's law to the internal temperature of the body. The two cannot be same unless the body is in thermal equilibrium with the surroundings. The temperature of the surface of the cooling body depends upon the strength of the draught of air. Even when the air convection is only due to heating of the layer in contact with the solid surface the temperature is much lower than the temperature inside the body. In fact if there is no gradient, there won't be cooling at all. When there is forced convection as in the case of presence of air draught, the temperature of the surface falls further because the gradient is larger. Since radiation loss depends on the temperature of the surface the faster cooling cannot be due to radiation. From all this discussion it is clear that Newton's law of cooling is completely different from Stefan's law of radiation and the derivation given in many Indian textbooks is erroneous. The heat loss in cooling is not radiation loss as is often mentioned in these textbooks. Also, it is not true that Newton's law of cooling is valid for only small temperature differences.

The law is accepted as an empirical one and experimentally well confirmed provided the cooling of the body is by convection currents at its surface. So one can infer that the law is related to the transfer of heat at the interface of a solid and a fluid. The heat conducted through the solid towards its surface is carried away by the currents of the fluid over its surface. If the hot body is kept in a closed chamber to protect from breeze etc., the cooling will be slower but still it depends upon the rate of convection of the air at the surface of the body. Further, to maintain the temperature of the air constant, it has to be surrounded by cold water so that heat flows from air to the surrounding water. If this arrangement is not made the temperature of the surrounding air would increase. The experiment can also be performed by placing the hot body in large amount of a liquid. Cooling the body under a fan or a liquid flowing over the body should also give results as expected. The only condition is that the rate of forced convection must be constant throughout the experiment. With forced convection the cooling will be much faster.

Let us see now the process of cooling of a solid suspended in a fluid. When a body with mass m and specific heat s cools, the rate of loss of heat is given by

$$\frac{dQ}{dt} = -ms\frac{d\theta}{dt},$$

where θ is the temperature of the body. But when a solid cools its temperature has a gradient from its centre to its surface. Hence there is no unique temperature of the body during the cooling process. Perhaps that is the reason why Newton's law of cooling is studied by observing the temperature fall of some liquid kept in a container. (The temperature of a liquid can be made uniform throughout its mass by stirring it.) During cooling process there has to be some temperature gradient across the wall of the container and the conduction through the wall leads to the loss of heat of the liquid. If K be the conductivity of the material of the container and A and d the area and thickness of the wall of the container, the rate of conduction of heat across this thickness is given by

$$\frac{dQ}{dt} = KA\frac{(\theta - \theta_0)}{d},$$

where θ and θ_0 are the temperatures of the inner and outer surfaces of the container respectively.

The heat lost by the body by conduction is equal to the heat carried away by the fluid in convection. Hence

$$ms\frac{d\theta}{dt} = -KA\frac{(\theta - \theta_0)}{d}$$

Therefore,

$$\frac{d\theta}{dt} = -\frac{KA}{msd}(\theta - \theta_0)$$

This appears similar to Newton's law of cooling because $\frac{KA}{msd}$ is constant. But the constant determined from experimental data is found to be much smaller than what we get by substituting the values of the constants. The reason is that neither the temperature of the inner surface of the container is the same as that of the liquid in the container nor the temperature of the outer surface is the same as that of the surrounding. That the outer surface is at a higher temperature than that of the surroundings is easily observed by simply touching the surface. Similarly we can infer that the temperature of the inner surface of the container must be lower than that of the liquid in the container. This phenomenon is observed whenever heat transfer takes place at a solid-fluid interface. There is a steep gradient of temperature across a very thin layer of the fluid in contact with the solid whenever there is heat transfer if the conductivity of the fluid is less than that of the solid. When there are convection currents in the fluid at the surfaces of solids the supply as well as removal of heat from the surface is controlled by the rate of convection. If there is forced convection, then cooling is faster because larger number of molecules of the cold fluid comes in contact with the outer surface of the solid and more heat is conducted from the solid surface to these molecules. In effect the surface of the fluid layer in contact with the solid to which heat is conducted is increased by forced convection. But what is important to note is that the temperature gradient across the wall of the container is less than $\frac{\theta - \theta_0}{d}$. Alternately, one may assume that the thickness d is increased to 'effective thickness' d_{eff} due to convection effects.

The discussion above clearly shows that Newton's law of cooling is a consequence of heat conduction (outwards from a body), whose rate is controlled by the convection processes at the surfaces. The loss of heat due to radiation is negligible as compared to the loss due to conduction-convection process. There are many examples to demonstrate this. The loss of heat from vacuum flasks is due to only radiation. In double walled plastic flasks used to store cold water, the air trapped between the walls being a poor conductor, in the absence of convection very little heat is conducted from the outer wall to the inner wall. But heat radiation process is not obstructed. This means the transfer of heat by radiation in this case is negligible.

The analogical processes in other fields where equations similar to that of Newton's law of cooling are obtained include: 1) fall of voltage across a capacitor being discharged through a resistive path and 2) fall of level of a viscous liquid in a container when the liquid is made to flow out through a tube. Both these processes are dependent on the relevant conductivity (or resistivity) of the conductor. The conclusion is that Newton's law of cooling is a consequence of conduction and not radiation.

The curve of voltage versus time during the discharge of a capacitor is similar to the temperature difference versus time curve of a cooling body. Hence one can expect a curve similar to that of the charging capacitor when a body is heated by placing it in a constant temperature environment. Such experiment when performed does confirm the conjecture. What this means is that the processes of heating and cooling bodies with environment at constant temperature are governed by exponential relationships similar to those of charging and discharging a capacitor. Newton's Law of Cooling describes only the cooling process.

To study the law experimentally in a better way we can write the expression in a different form. Since θ_0 is constant $d\theta_0/dt = 0$. So

$$ms\left[\frac{d\theta}{dt} - \frac{d\theta_0}{dt}\right] = -KA\frac{(\theta - \theta_0)}{d} \text{ and}$$

$$\left[\frac{d\theta}{dt} - \frac{d\theta_0}{dt}\right] = -\frac{KA}{msd}dt$$

As mentioned above the factor (KA/msd) must be replaced by a constant taking into account the fact that the 'effective thickness' is much larger than d due to the convection effects. If the rate of convection is constant, then representing this constant by μ and integrating the above expression, we obtain

$$\ln(\theta - \theta_0) = \text{const} - \mu t$$

Thus the graph of logarithm of temperature difference versus time is a straight line. To see that the constant μ happens to be same for both heating as well as cooling processes, the following experiment can be performed. A metallic container containing water at room temperature may be suspended in a large vessel containing boiling water. The rise in the temperature of water in the metallic container is noted with time. The graph of logarithm of temperature difference against time can then be plotted and compared with similar graph obtained when the same amount of hot water in the container is allowed to cool by placing the container in a large vessel with water at room temperature.

1) B.L.Worsnop and H.T.Flint, *Advanced Practical Physics for Students* Ninth Edition, Macmillan, London, p.264.